n−dimensional spheres and spherical shells

The volume of an $n-$dimensional sphere of radius R is given by

$$V(R) = \frac{\pi^{n/2}}{\Gamma\left(n/2+1\right)} R^n.$$

So, for example, the volume of a disk (its area) is given by

$$V(R) = \pi R^2$$

of a $3-$dimensional sphere it is

$$V(R) = \frac{4}{3}\pi R^3.$$

and so forth.

If we want to compute the fraction of volume that is contained in a spherical shell defined by $(1-\epsilon)R \leq r \leq R$ where ϵ is small, say 0.01 or 0.001 we get

$$\delta V(R) = V(R) - V((1-\epsilon)R)$$

The fraction of volume is given by

$$\begin{aligned}\frac{\delta V(R)}{V(R)} &= 1 - \frac{V((1-\epsilon R))}{V(R)} \\ &= 1 - \left[\frac{(1-\epsilon)R}{R}\right]^n \\ &= 1 - \left[1-\epsilon\right]^n.\end{aligned}$$

What does this mean?

Say you have a 3d-sphere of radius $R = 1$m and you take a shell of thickness 1cm which is 1% of the radius. This shell contains about 3% of the volume/mass of the original sphere. But if you have a 100d-sphere, the shell contains already 64% of the mass.

In statistical physics, n is often the number of degrees of freedom of a system, e.g. moving particles in a box so a number of the order of Avogadro's number like 10^{23}. This means, regardless of how tiny ϵ that second term $[1-\epsilon]^n$ is negligible and effectively everything is happening on the shell.